KC-8047

B. E. - II (Sem. III) (All) Examination

November / December – 2012

Engineering Mathematics - III

(New Syllabus)

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. - 2 (Sem. 3) (All)**

Name of the Subject :
☛ **ENGINEERING MATHEMATICS - 3 (NEW)**

☛ Subject Code No. : **8 0 4 7** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures on the right indicate marks.
(4) Draw the figure whenever it is necessary.

1 (a) Attempt the following : **10**

(1) Find the value of λ for which the equation.

$$\left(xy^2+\lambda x^2 y\right)dx+\left(x+y\right)dy=0 \text{ is exact.}$$

(2) Use method of reduction to find the second L.I. solution of $y''+6y'+9y=0$ given that $y_1(x)=e^{-3x}$ is one solution.

(3) Find the complementary function of $y''+4y'+4y=e^x$.

(4) Define first order and first degree linear differential equation.

(5) Evaluate $\int_{-1}^{1} P_2^2(x)\,dx$.

(b) Attempt any two of the following : **10**

(1) Find power series solution of $y''+y=0$.

(2) Obtain a Frobenius series solution of $4xy''+2(1-x)y'-y=0$.

(3) Use Frobenius series solution method to obtain the solution of $x^2y''+xy'+\left(x^2-4\right)y=0$.

2 (a) Attempt any two of the following : **6**

(1) Solve $xdy-ydx=\sqrt{x^2+y^2}$

(2) Solve $\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dx$

(3) Solve $(x+y+1)^2\frac{dy}{dx}=1$.

(b) Attempt any three of the following : **9**

(1) Solve $y''+3y'+2y=5$

(2) $x^2y''-4xy'+6y=21x^{-4}$

(3) Solve using Method of variation of parameters, $y''+y=\operatorname{cos}ecx$.

(4) Use method of undetermined coefficient to solve $y''+4y=8x^2$.

3 (a) Attempt any three of the following : **9**

(1) $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.

(2) Prove that $\int_0^\infty e^{-ax}x^{n-1}dx=\frac{\Gamma n}{d^n}$.

(3) Examine whether $e^{2x}, xe^{2x}, x^2e^{2x}$ are linearly independent.

(4) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx=\frac{\beta(m,n)}{a^nb^m}$

(b) Attempt any two of the following : **6**

(1) Find Fourier sine integral of $f(x)=\begin{cases}\sin x & ;0\le x\le\pi\\ 0 & ;\ x>\pi\end{cases}$

(2) Find the Fourier transform of $f(x)=xe^{-x^2}$.

(3) Find Fourier cosine transform of

$$f(x)=\begin{cases}k & if\ \ 0<x<a\\ 0 & if\ \ x>a\end{cases}$$

4 (a) Attempt the following : **10**

(1) State second shifting theorem and using it evaluate, $L\left[4\sin(t-3)u(t-3)\right]$.

(2) Find $L^{-1}\left(\frac{1}{2s-3}\right)$.

(3) State one dimensional heat equation and state which method we apply to find the solution of it.

(4) Evaluate $L\left(t^3\right)$.

(5) Define finite Fourier sine and cosine transforms.

(b) Solve any two following using Laplace transform technique : **10**

(1) $y''+y=e^{-1},\ y(0)=y_0,\ y'=(0)=y_1$

(2) Using convolution theorem evaluate $L^{-1}\left(\frac{1}{(s-1)(s+2)}\right)$

(3) $y''-2y'-8y=0,\ y(0)=3,\ y'(0)=6$

5 (a) Attempt any two of the following : **10**

(1) Obtain the Fourier series to represent $f(x)=\frac{1}{4}(\pi-x)^2, 0<x<2\pi$.

(2) If $f(x)=x^2$ for $-\pi<x<\pi$ and hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

(3) Obtain Fourier series expansion for $f(x) = \frac{1}{2}(\pi - x)$ for $0 < x < 2$.

(b) Attempt any two of the following : **6**

(1) Find Laplace transform of $\sin^2 3t$.

(2) Find Laplace transform of $e^{-3t}(\cos 4t + 3\sin 4t)$.

(3) Find $L^{-1}\left[\frac{1}{s(s+1)}\right]$.

6 (a) Solve the following one-dimensional Heat equation **7**

$u_{tt} = c^2 u_{xx}; t > 0$ and $0 < x < l$

$u(x, 0) = f(x);\ t > 0$

$u_t(x, 0) = 0$

(b) Attempt any one of the following : **7**

(1) Find the deflection $u(x, t)$ of the vibrating string of length π and its ends are fixed, corresponding to zero initial velocity and the initial deflection is $u(x, 0) = 2(\sin x + \sin 3x)$.

(2) Solve the boundary value problem, $u_{xx} + u_{yy} = 0$ conditions are,

$u(x, 0) = u(0, y) = u(l, y) = 0$ and

$u(x, 0) = \frac{\sin n\pi x}{l}; 0 \le x \le l$ and $0 \le y \le a$.
